Unit I

INTRODUCTION TO LINEAR PROGRAMMING PROBLEM FORMULATION & GRAPHICAL SOLUTION OF LPP

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•Decision variables:

mathematical symbols representing levels of activity of a firm.

•Objective function:

linear mathematical relationship describing an objective of the firm, in terms of decision variables, that is maximized or minimized

•Constraints:

restrictions placed on the firm by the operating environment stated in linear relationships of the decision variables.

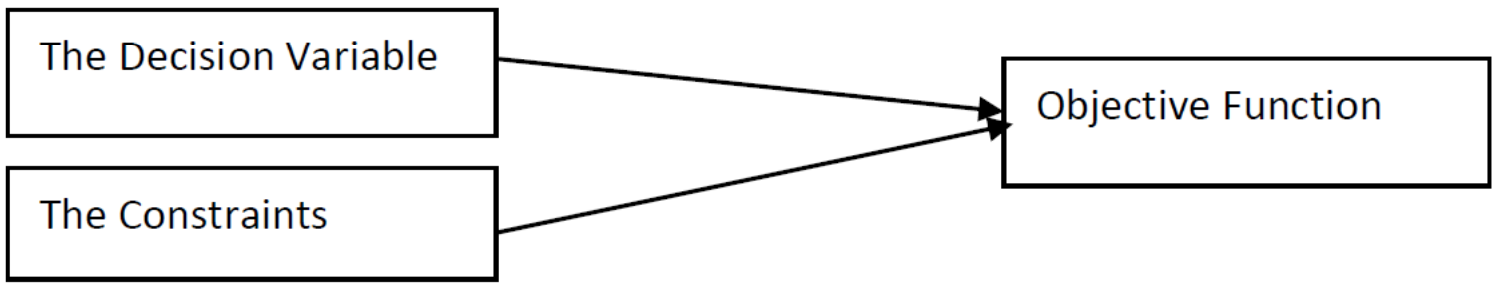
•Parameters:

numerical coefficients and constants used in the objective function and constraint equations.

FORMULATION OF THE LINEAR PROGRAMMING MODEL

Three components are:

1. The decision variable
2. The environment (uncontrollable) parameters
3. The result (dependent) variable



Linear Programming Model is composed of the same components

Mathematical Formulation of L.P.P

If x_j ($j = 1, 2, \ldots, n$) are the n decision variables of the problem and if the system is subject to m constraints, the general Mathematical model can be written in the form :

$$\text{Optimize } Z = f(x_1, x_2, \ldots x_n)$$

$$\text{subject to } g_i(x_1, x_2, \ldots x_n) \quad \leq, =, \geq b_i, \; (i = 1, 2, \ldots, m)$$

(called structural constraints)

$$\text{and} \quad x_1, x_2, \ldots x_n \geq 0,$$

(called the non–negativity restrictions or constraints)

Procedure for forming a LPP Model

Step 1 : Identify the unknown decision variables to be determined and assign symbols to them.

Step 2 : Identify all the restrictions or constraints (or influencing factors) in the problem and express them as linear equations or inequalities of decision variables.

Step 3 : Identify the objective or aim and represent it also as a linear function of decision variables.

Step 4 : Express the complete formulation of LPP as a general mathematical model.

We consider only those situations where this will help the reader to put proper inequalities in the formulation.

1. Usage of manpower, time, raw materials etc are always less than or equal to the availability of manpower, time, raw materials etc.

2. Production is always greater than or equal to the requirement so as to meet the demand.

Example 1: Suppose an industry is manufacturing two types of products P1 and P2. The profits per Kg of the two products are Rs.30 and Rs.40 respectively. These two products require processing in three types of machines. The following table shows the available machine hours per day and the time required on each machine to produce one Kg of P1 and P2. Formulate the problem in the form of linear programming model.

Profit/Kg	P1 Rs.30	P2 Rs.40	Total available Machine hours/day
Machine 1	3	2	600
Machine 2	3	5	800
Machine 3	5	6	1100

Solution:

The procedure for linear programming problem formulation is as follows:

Introduce the decision variable as follows:
Let x_1 = amount of P1
x_2 = amount of P2

In order to maximize profits, we establish the objective function as

$$30x_1 + 40x_2$$

Since one Kg of P1 requires 3 hours of processing time in machine 1 while the corresponding requirement of P2 is 2 hours. So, the first constraint can be expressed as

$$3x_1 + 2x_2 \leq 600$$

Similarly, corresponding to machine 2 and 3 the constraints are

$$3x_1 + 5x_2 \leq 800$$
$$5x_1 + 6x_2 \leq 1100$$

In addition to the above there is no negative production, which may be represented algebraically as

$$x_1 \geq 0 \quad ; \quad x_2 \geq 0$$

Thus, the product mix problem in the linear programming model is as follows:

Maximize
$$30x_1 + 40x_2$$

Subject to:
$$3x_1 + 2x_2 \leq 600$$
$$3x_1 + 5x_2 \leq 800$$
$$5x_1 + 6x_2 \leq 1100$$
$$x_1 \geq 0,\ x_2 \geq 0$$

Example 2

A company owns two flour mills viz. A and B, which have different production capacities for high, medium and low quality flour. The company has entered a contract to supply flour to a firm every month with at least 8, 12 and 24 quintals of high, medium and low quality respectively. It costs the company Rs.2000 and Rs.1500 per day to run mill A and B respectively. On a day, Mill A produces 6, 2 and 4 quintals of high, medium and low quality flour, Mill B produces 2, 4 and 12 quintals of high, medium and low quality flour respectively. How many days per month should each mill be operated in order to meet the contract order most economically.

Solution:

Let us define x1 and x2 are the mills A and B. Here the objective is to minimize the cost of the machine runs and to satisfy the contract order. The linear programming problem is given by

Minimize

$$2000x_1 + 1500x_2$$

Subject to:

$$6x_1 + 2x_2 \geq 8$$
$$2x_1 + 4x_2 \geq 12$$
$$4x_1 + 12x_2 \geq 24$$

$$x_1 \geq 0, x_2 \geq 0$$

Example 3

Four different type of metals, namely, iron, copper, zinc and manganese are required to produce commodities A, B and C. To produce one unit of A, 40kg iron, 30kg copper, 7kg zinc and 4kg manganese are needed. Similarly, to produce one unit of B, 70kg iron, 14kg copper and 9kg manganese are needed and for producing one unit of C, 50kg iron, 18kg copper and 8kg zinc are required. The total available quantities of metals are 1 metric ton iron, 5 quintals copper, 2 quintals of zinc and manganese each. The profits are Rs 300, Rs 200 and Rs 100 by selling one unit of A, B and C respectively. Formulate the problem mathematically. Solution: Let z be the total profit and the problem is to maximize z (called the objective function). We write below the given data in a tabular form

	Iron	Copper	Zinc	Manganese	Profit per unit in Rs
A	40kg	30kg	7kg	4kg	300
B	70kg	14kg	0kg	9kg	200
C	60kg	18kg	8kg	0kg	100
Available quantities→	1000kg	500kg	200kg	200kg	

Solution:

To get maximum profit, suppose x_1 units of A, x_2 units of B and x_3 units of C are to be produced. Then the total quantity of iron needed is $(40x_1 + 70x_2 + 60x_3)$kg.

Similarly, the total quantity of copper, zinc and manganese needed are $(30x_1 + 14x_2 + 18x_3)$kg , $(7x_1 + 0x_2 + 8x_3)$kg and $(4x_1 + 9x_2 + 0x_3)$kg respectively.

From the conditions of the problem we have,

The objective function is $z = 300x_1 + 200x_2 + 100x_3$ which is to be maximized. Hence the problem can be formulated as,

Maximize $z = 300x_1 + 200x_2 + 100x_3$

Subject to $40x_1 + 70x_2 + 60x_3 \leq 1000$

$$30x_1 + 14x_2 + 18x_3 \leq 500$$

$$7x_1 + 0x2 + 8x_3 \leq 200$$

$$4x_1 + 9x_2 + 0x_3 \leq 200$$

As none of the commodities produced can be negative, $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$. All these inequalities are known as constraints or restrictions.

Example 4

A patient needs daily 5mg, 20mg and 15mg of vitamins A, B and C respectively. The vitamins available from a mango are 0.5mg of A, 1mg of B, 1mg of C, that from an orange is 2mg of B, 3mg of C and that from an apple is 0.5mg of A, 3mg of B, 1mg of C. If the cost of a mango, an orange and an apple be Rs 0.50, Rs 0.25 and Rs 0.40 respectively, find the minimum cost of buying the fruits so that the daily requirement of the patient be met. Formulate the problem mathematically.

Solution:

The problem is to find the minimum cost of buying the fruits. Let z be the objective function. Let the number of mangoes, oranges and apples to be bought so that the cost is minimum and to get the minimum daily requirement of the vitamins be x_1, x_2, x_3 respectively. Then the objective function is given by

$z = 0.50\ x_1 + 0.25\ x_2 + 0.40\ x_3$

From the conditions of the problem

$0.5x_1 + 0x_2 + 0.5x_3 \geq 5$

$x_1 + 2x_2 + 3x_3 \geq 2$

$x_1 + 3x_2 + x_3 \geq 15$ and $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Hence the problem is

Minimize $z = 0.50\, x_1 + 0.25\, x_2 + 0.40\, x_3.$

Subject to

$$0.5x_1 + 0x_2 + 0.5x_3 \geq 5$$
$$x_1 + 2x_2 + 3x_3 \geq 20$$
$$x_1 + 3x_2 + x_3 \geq 15$$

and $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

GRAPHICAL SOLUTION OF LINEAR PROGRAMMING MODELS

- Graphical solution is limited to linear programming models containing only two decision variables. (Can be used with three variables but only with great difficulty.)

- Graphical methods provide visualization of how a solution for a linear programming problem is obtained.

METHODOLOGY OF GRAPHICAL METHOD

- Step 1: Formulation of LPP (Linear Programming Problem) Use the given data to formulate the LPP.
- Step 2: Determination of each axis Horizontal (X) axis: Product A (X1) Vertical (Y) axis: Product B (X2).
- Step 3: Finding co-ordinates of constraint lines to represent constraint lines on the graph. The constraints are presently in the form of inequality (≤). We should convert them into equality to obtain co-ordinates.
- Step 4: Representing constraint lines on graph To mark the points on the graph, we need to select appropriate scale. Which scale to take will depend on maximum value of X1 & X2 from co-ordinates.
- Step 5: Identification of Feasible Region The feasible region is the region bounded by constraint lines. All points inside the feasible region or on the boundary of the feasible region or at the corner of the feasible region satisfy all constraints.
- Step 6: Finding the optimal Solution The optimal solution always lies at one of the vertices or corners of the feasible region.

PROBLEM1. Solve graphically the given linear programming problem.

$$\text{Maximize } Z = 40x_1 + 50x_2$$

subject to

$$1x_1 + 2x_2 \leq 40$$

$$4x_1 + 3x_2 \leq 120$$

$$x_1, x_2 \geq 0$$

Solution: Considering the constraint inequality and plotting then in graph we have:

Let constraint 1 = $1x_1 + 2x_2 \leq 40$

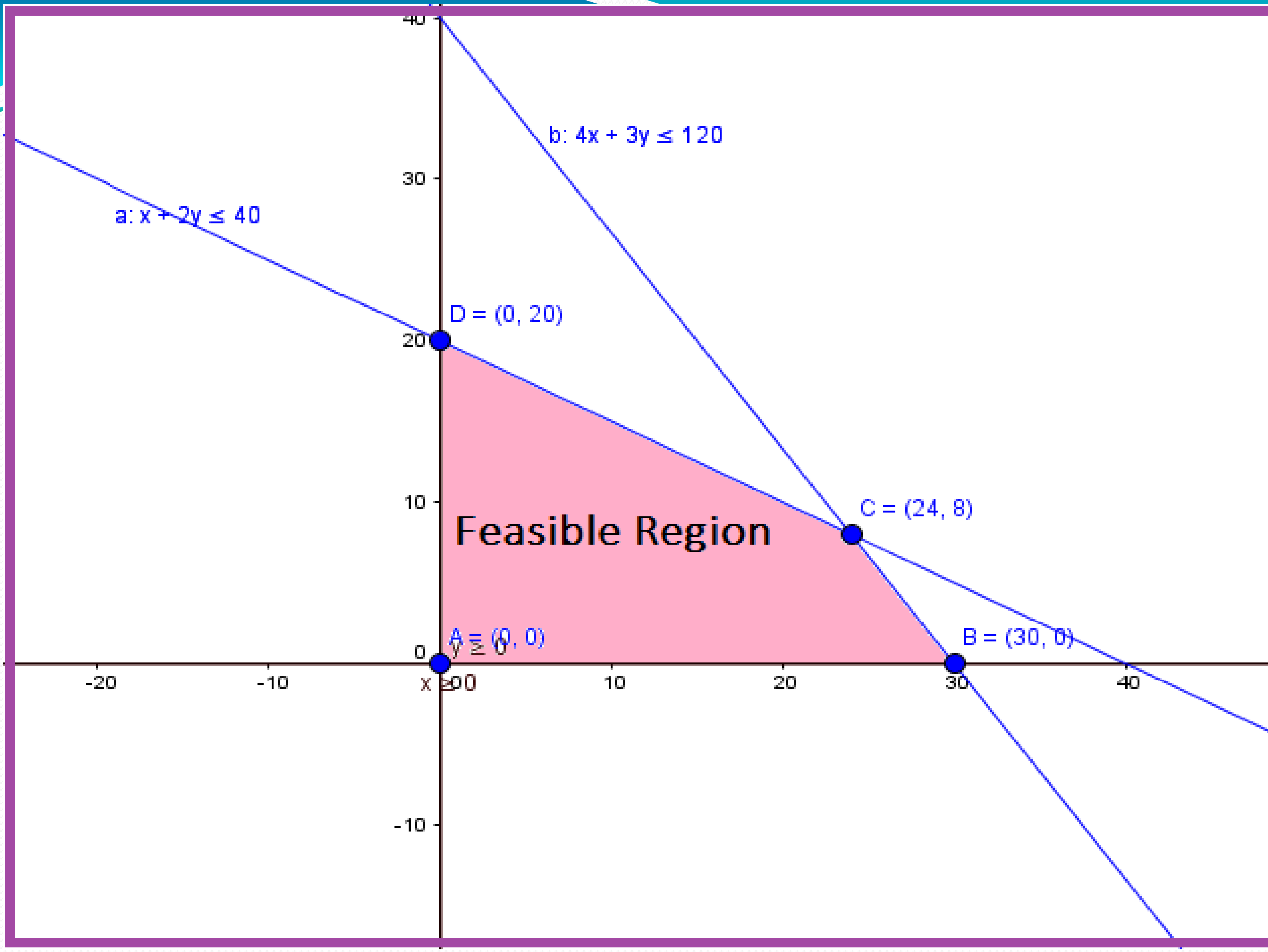
That is $1x_1 + 2x_2 = 40$ gives (0,20) and (40,0).

Let constraint 2 = $4x_1 + 3x_2 \leq 120$

That is $4x1 + 3x_2 \leq 120$ gives (0,40) and (30,0)

Let constraint 3 = $x_1, x_2 \geq 0$

That is $x_1 = 0$, $x_2 = 0$


b: 4x + 3y ≤ 120
a: x + 2y ≤ 40
D = (0, 20)
C = (24, 8)
Feasible Region
A = (0, 0)
B = (30, 0)
y ≥ 0
x ≥ 0
40
30
20
10
0
-10
-20
-10
10
20
30
40

From the graph the Feasible region is as ABCD

S. No	Point	Z= $40x_1 + 50x_2$
1	A(0,0)	0
2	B(30,0)	1200
3	C(24,8)	1360 = Max z
4	D(0,20)	1000

Therefore, the Max Z = 1360 , x_1 =24 and x_2 =8

PROBLEM 2 . Solve graphically the given linear programming problem.

Minimize $Z= 3x_1 + 5x_2$

subject to

$-3x_1 + 4x_2 \leq 12$

$x1 \leq 4$

$2x_1 - x_2 \geq -2$

$x_2 \geq 2$

$2x_1 + 3x_2 \geq 12$

$x_1, x_2 \geq 0$

Solution: Considering the constraint inequality and plotting then in graph we have:

Let constraint 1 = $-3x_1 + 4x_2 \leq 12$: (0,3) and (-4,0)

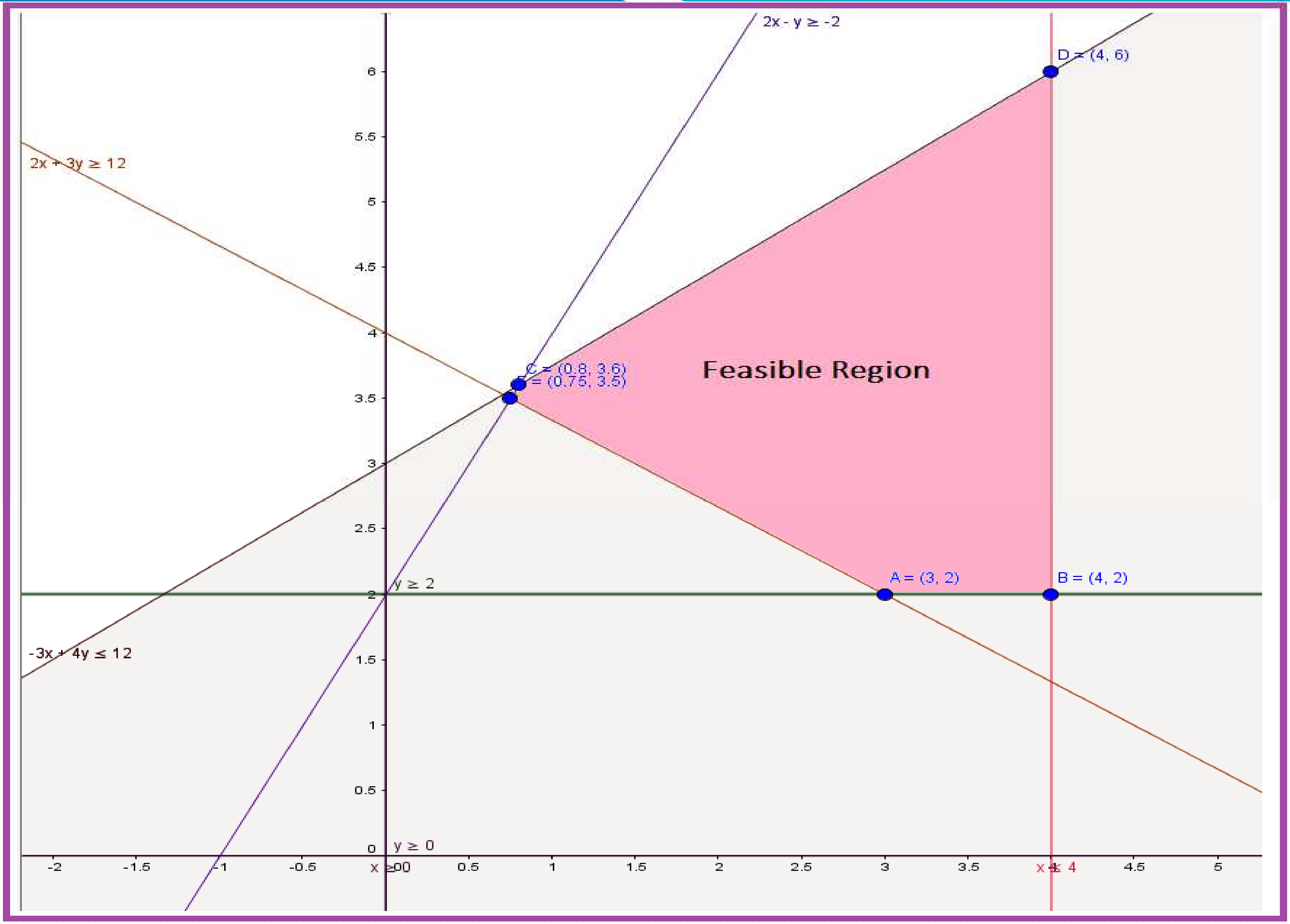
Let constraint 2 = $x1 \leq 4$: $x1=4$

Let constraint 3 = $2x_1 - x_2 \geq -2$: (0,2) and (-1,0)

Let constraint 4 = $x_2 \geq 2$: $x_2 = 2$

Let constraint 5 = $2x_1 + 3x_2 \geq 12$: (0,4) and (6,0)

Let constraint 5 = $x_1, x_2 \geq 0$:$x_1 = 0$, $x_2 = 0$


2x - y ≥ -2
D = (4, 6)
2x + 3y ≥ 12
C = (0.8, 3.6)
E = (0.75, 3.5)
Feasible Region
A = (3, 2)
B = (4, 2)
y ≥ 2
-3x + 4y ≤ 12
y ≥ 0
x ≥ 0
x ≤ 4

From the graph the Feasible region is as ABDCE

S. No	Point	Z= $3x_1 + 5x_2$
1	A(3,2)	19 = Min Z
2	B(4,2)	22
3	C(4/5,18/5)	102/5
4	D(4,6)	42
5	E(3/4,7/2)	79/4

Therefore, the Min Z = 19 , x_1 =3 and x_2 =2

PROBLEM 3 . Solve graphically the given linear programming problem.

$$\text{Maximize } Z = x_1 - 2x_2$$

subject to

$$-x_1 + x_2 \leq 1$$

$$6x_1 + 4x_2 \geq 24$$

$$0 \leq x_1 \leq 5$$

$$2 \leq x_2 \leq 4$$

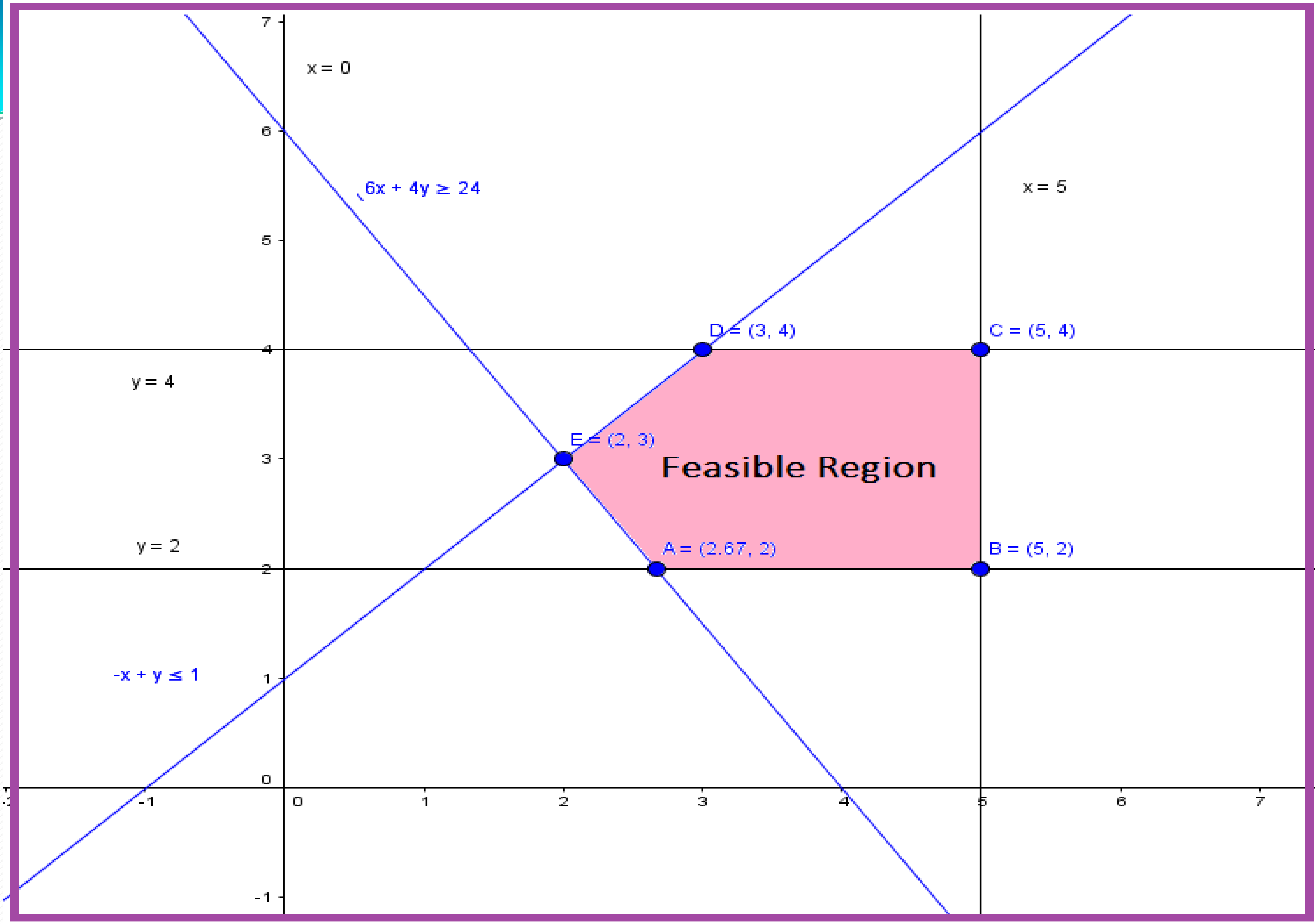
Solution: Considering the constraint inequality and plotting then in graph we have:

Let constraint 1 = $-x_1 + x_2 \leq 1$: (0,1) and (-1,0)

Let constraint 2 = $6x_1 + 4x_2 \geq 24$: (0,6) and (4,0)

Let constraint 3 = $0 \leq x_1 \leq 5$: $x_1 = 0$ and $x_1 = 5$

Let constraint 4 = $2 \leq x_2 \leq 4$: $x_2 = 2$ and $x_2 = 4$


x = 0
6x + 4y ≥ 24
x = 5
D = (3, 4)
C = (5, 4)
y = 4
E = (2, 3)
Feasible Region
y = 2
A = (2.67, 2)
B = (5, 2)
-x + y ≤ 1

From the graph the Feasible region is as ABCDE

S. No	Point	Z= $x_1 - 2x_2$
1	A(8/3,2)	-4/3
2	B(5,2)	1 = Max Z
3	C(5,4)	-3
4	D(3,4)	-5
5	E(2,3)	-4

Therefore, the Max Z = 1 , $x_1 = 5$ and $x_2 = 2$

PROBLEM 4. Solve graphically the given linear programming problem.

$$\text{Maximize } Z = 2x_1 + 3x_2$$

subject to

$$x_1 - x_2 \leq 2$$

$$x_1 + x_2 \geq 4$$

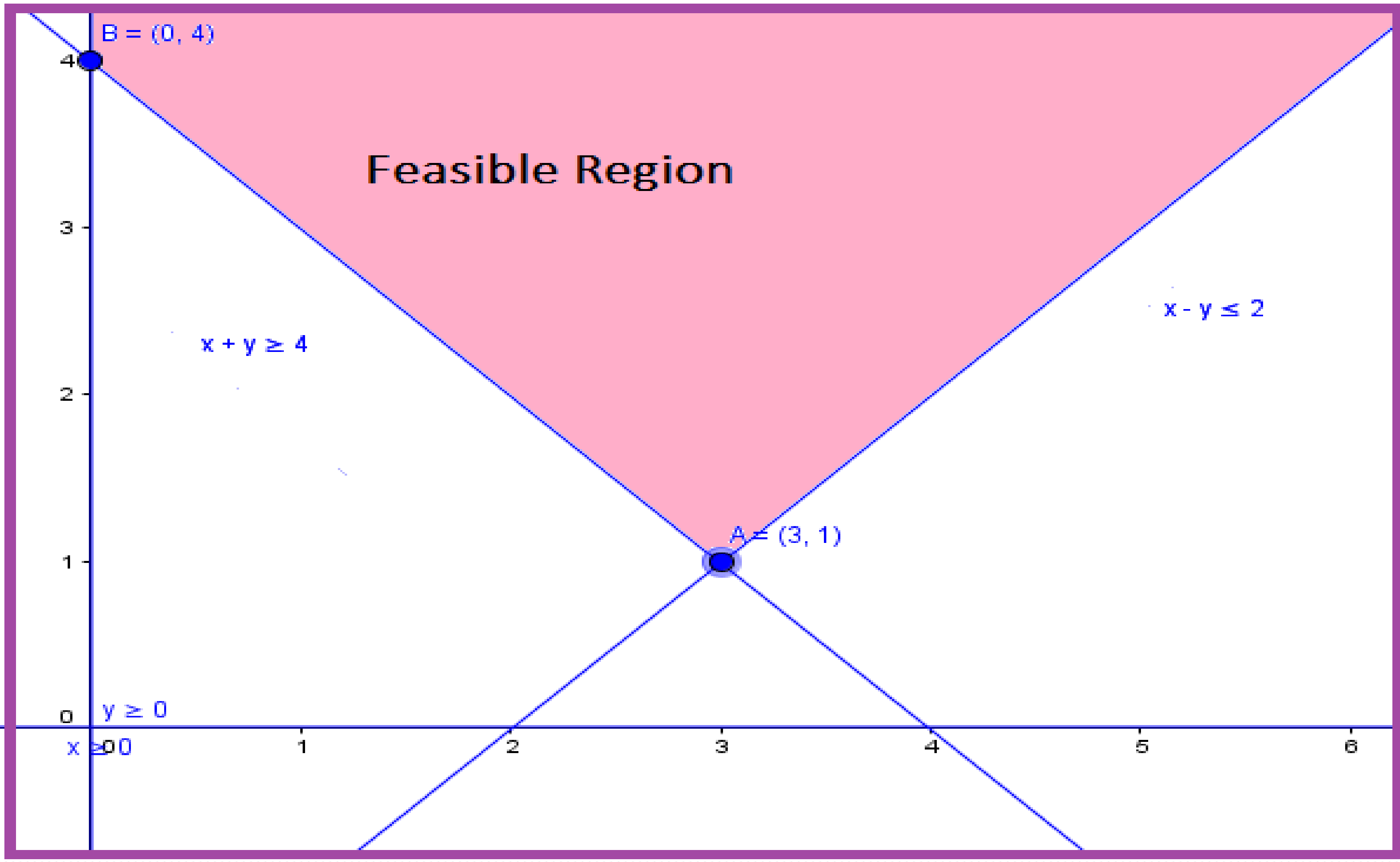
$$x_1, x_2 \geq 0$$

Solution: Considering the constraint inequality and plotting then in graph we have:

Let constraint 1 = $x_1 - x_2 \leq 2$: (0, -2) and (2,0)

Let constraint 2 = $x_1 + x_2 \geq 4$: (0,4) and (4,0)

Let constraint 3 = $x_1, x_2 \geq 0$: $x_1 = 0$, $x_2 = 0$



Here the solution space is unbounded.
So considering the point on the feasible region A(3,1) and B(0,4) the max z=12 at B.

PROBLEM 5. Solve graphically the given linear programming problem.

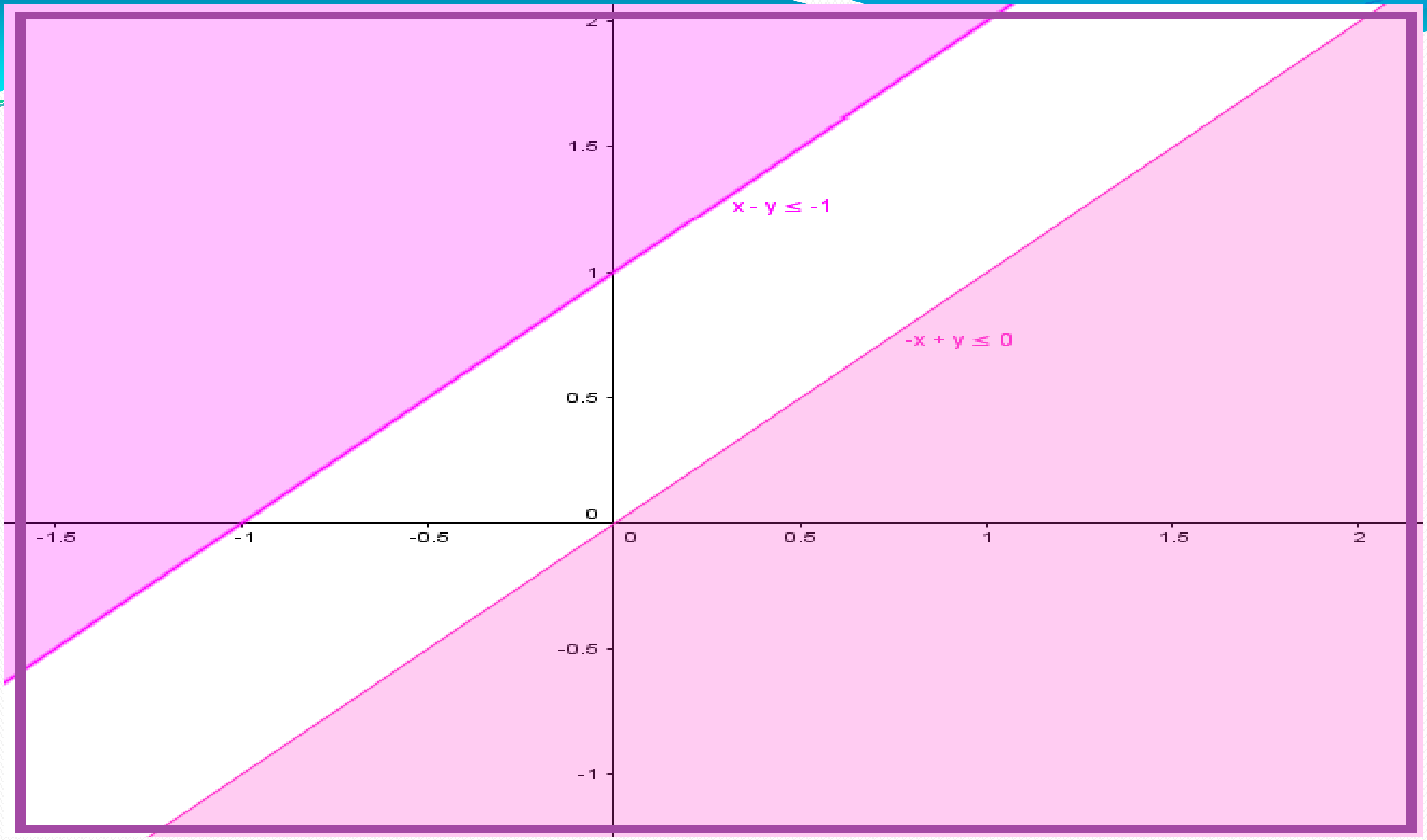
$$\text{Maximize } Z = 4x_1 + 3x_2$$
subject to
$$x_1 - x_2 \leq -1$$
$$-x_1 + x_2 \leq 0$$
$$x_1, x_2 \geq 0$$

Solution: Considering the constraint inequality and plotting then in graph we have:

Let constraint 1 = $x_1 - x_2 \leq -1$: (0, 1) and (-1,0)

Let constraint 2 = $-x_1 + x_2 \leq 0$: $x_2 \leq x_1$

Let constraint 3 = $x_1, x_2 \geq 0$: $x_1 = 0$, $x_2 = 0$



Here there no feasible region exist. That is no solution space.

THANK YOU

SIMPLEX METHOD - PRACTICE PROBLEMS

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Problem 1:

$$\text{Max. } Z = 13x_1 + 11x_2$$

Subject to constraints:

$$4x_1 + 5x_2 \leq 1500$$

$$5x_1 + 3x_2 \leq 1575$$

$$x_1 + 2x_2 \leq 420$$

$$x_1,\ x_2 \geq 0$$

Solution :

- **Step 1:** Convert all the inequality constraints into equalities by the use of slack variables

Let S_1, S_2, S_3 be three slack variables.

Introducing these slack variables into the inequality constraints and rewriting the objective function such that all variables are on the left-hand side of the equation. Model can rewritten as:

$$Z - 13x_1 - 11x_2 = 0$$

Subject to constraints:

$$4x_1 + 5x_2 + S_1 = 1500$$

$$5x_1 + 3x_2 + S_2 = 1575$$

$$x_1 + 2x_2 + S_3 = 420$$

$$x_1, x_2, s_1, s_2, s_3 \geq 0$$

* **Step II**: Find the Initial BFS.

One Feasible solution that satisfies all the constraints is: $x_1 = 0$, $x_2 = 0$, $S_1 = 1500$, $S_2 = 1575$, $S_3 = 420$ and $Z=0$.

Now, S_1, S_2, S_3 are Basic variables.

Row NO.	Basic Variable	Coefficients of:						Sol.	Ratio
		Z	x_1	x_2	S_1	S_2	S_3		
A1	Z	1	-13	-11	0	0	0	0	
B1	S_1	0	4	5	1	0	0	1500	375
C1	S_2	0	5	3	0	1	0	1575	315
D1	S_3	0	1	2	0	0	1	420	420

Step IV: a) Choose the most negative number from row A1(i.e Z row). Therefore, x_1 is a ***entering variable***.

Step V: x_1 becomes ***basic variable*** and S_2 becomes ***non basic variable.*** New table is:

Row NO.	Basic Variable	Coefficients of:						Sol.	Ratio
		Z	x_1	x_2	S_1	S_2	S_3		
A1	Z	1	0	-16/5	0	13/5	0	4095	
B1	S_1	0	0	13/5	1	-4/5	0	240	92.3
C1	x_1	0	1	3/5	0	1/5	0	315	525
D1	S_3	0	0	7/5	0	-1/5	1	105	75

Next Table is :

Row NO.	Basic Variable	Coefficients of:						Sol.
		Z	x_1	x_2	S_1	S_2	S_3	
A1	Z	1	0	0	0	15/7	16/7	4335
B1	S_1	0	0	0	1	-3/7	-13/7	45
C1	x_1	0	1	0	0	2/7	-3/7	270
D1	x_2	0	0	1	0	-1/7	5/7	75

Optimal Solution is : **x_1= 270, x_2= 75, Z= 4335**

Problem 2:

2. Solve the LPP by simplex method.

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

Subject to $x_1 + 4x_2 \leq 420$

$3x_1 + 2x_3 \leq 460$

$x_1 + 2x_2 + x_3 \leq 430$

Solution :

By introducing non-negative slack variables s_1, s_2 and s_3 the standard form of LPP becomes

Maximize $Z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to $x_1 + 4x_2 + s_1 + 0s_2 + 0s_3 = 420$

$3x_1 + 2x_3 + 0s_1 + s_2 + 0s_3 = 460$

$x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 430$

And $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Since there are 3 equations with 5 variables, the initial basic feasible solution is obtained by (6-3)=3 variables to zero.

The IBFS is $(x_1 = 0, x_2 = 0, x_3 = 0,$ non-basic$), s_1 = 420, s_2 = 460, s_3 = 430$

The initial simplex table is

Initial Iteration:

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	ratio
0	s_1	420	1	4	0	1	0	0	-
0	s_2	460	3	0	(2)	0	1	0	460/2=230*
0	s_3	430	1	2	1	0	0	1	430/1=430
Z_j-C_j		0	-3	-2	-5	0	0	0	

Since there are some (Z_j-C_j) are negative, the current basic feasible solution is not optimal.

X3 enters and s2 leaves the basis.

First Iteration:

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	ratio
0	s_1	420	1	4	0	1	0	0	420/4=105
5	x_3	230	3/2	0	1	0	1/2	0	-
0	s_3	200	-1/2	2	0	0	-1/2	1	200/2=100*
Z_j-C_j		1150	9/2	-2	0	0	5/2	0	

Since there is an $(Z_j-C_j) = -2$, the current basic feasible solution is not optimal.

Here x2 enters and s3 leaves the basis.

Second Iteration:

C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	ratio
0	s_1	20	2	0	0	1	1	-2	
5	x_3	230	3/2	0	1	0	1/2	0	
2	x_2	100	-1/4	1	0	0	-1/4	1/2	
Z_j-C_j		1350	4	0	0	0	2	1	

Since there are some $(Z_j - C_j) \geq 0$, the current basic feasible solution is optimal.

The optimal solution is Max Z=1350, x1=0, x2=100, x3=230.

THANK YOU

Unit II

TRANSPORTATION PROBLEM – DEGENERATE SOLUTION

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PROBLEM 1: Solve the Transportation table.

Warehouse

	A	B	C	D	E	F	Available
Factory 1	9	12	9	6	9	10	5
Factory 2	7	3	7	7	5	5	6
Factory 3	6	5	9	11	3	11	2
Factory 4	6	8	11	2	2	10	9
Requirement	4	4	6	2	4	2	

Solution:

Step 1: Check whether the problem is balanced or not.

If the total sum of all the supply from sources is equal to the total sum of all the demands for destinations then the transportation problem is a balanced

Warehouse

		A	B	C	D	E	F	Available
Factory	1	9	12	9	6	9	10	5
	2	7	3	7	7	5	5	6
	3	6	5	9	11	3	11	2
	4	6	8	11	2	2	10	9
Requirement		4	4	6	2	4	2	22

Vogel's approximation method, the initial solution is as shown in the following table:

9	12	9 **5**	6	9	10
7	3 **4**	7	7	5	5 **2**
6 **1**	5	9 **1**	11	3	11
6 **3**	8	11	2 **2**	2 **4**	10

Since the number of non-negative allocations is 8 which is less than $(m + n - 1) = (4 + 6 - 1) = 9$, this basic solution is degenerate one.

To resolve degeneracy, we allocate a very small quantity $\in$ to the cell (4,2), so that the number of occupied cells becomes $(m + n - 1)$. Hence the non-degenerate basic feasible solution is as shown in the following table.

9	12	9 **5**	6	9	10
7	3 **4**	7	7	5	5 **2**
6 **1**	5	9 **1**	11	3	11
6 **3**	8 $\in$	11	2 **2**	2 **4**	10

The initial transportation cost = Rs. $9 \times 5 + 3 \times 4 + 5 \times 2 + 6 \times 1$

$+8 \times \in + 9 \times 1 + 6 \times 3 + 2 \times 2 + 2 \times 4$

= Rs. $(112 + 8 \in)$ = Rs. 112/- , $\in \to 0$.

	v1=6	v2=8	v3=9	v4=2	v5=2	v6=10
u1=0	9 6 3	12 8 4	9 **5**	6 2 4	9 2 7	10 10 0
u2=-5	7 1 6	3 **4**	7 4 3	7 -3 10	5 -3 8	5 **2**
u3=0	6 **1**	5 8 * -3	9 **1**	11 2 9	3 2 1	11 10 1
u4=0	6 **3**	8 ∈	11 9 2	2 **2**	2 **4**	10 10 0

Form a new BFS by giving maximum allocation to the cell for which d_{ij} is most negative by making an occupied cell empty. For that draw a closed path consisting of horizontal and vertical lines beginning and ending at the cell for which d_{ij} is most negative and having its other corners at some allocated cells. Along this closed loop indicate $+\theta$ and $-\theta$ alternatively at the corners. Choose minimum of the allocations from the cells having $-\theta$. Add this minimum allocation to the cells with $+\theta$ and subtract this minimum allocation from the allocation to the cells with $-\theta$.

9 6 3	12 8 4	9 **5**	6 2 4	9 2 7	10 10 0
7 1 6	3 **4**	7 4 3	7 -3 10	5 -3 8	5 **2**
6 – **1**	5 8 * + -3	9 **1**	11 2 9	3 2 1	11 10 1
6 + **3**	8 – ∈	11 9 2	2 **2**	2 **4**	10 10 0

	v1= 6	v2=5	v3=9	v4=2	v5=2	v6=7
u1=0	9 6 3	12 5 7	9 **5**	6 2 4	9 2 7	10 7 3
u2=-2	7 4 3	3 **4**	7 7 0	7 0 7	5 0 5	5 **2**
u3=0	6 **1**	5 **∈**	9 **1**	11 2 9	3 2 1	11 7 4
u4=0	6 **3**	8 5 3	11 9 2	2 **2**	2 **4**	10 7 3

Since all $d_{ij} > 0$ with $d_{23} = 0$, the solution under the test is optimal and an alternative optimal solution is also exists.

$\therefore$ The optimum allocation schedule is given by $x_{13} = 5$, $x_{22} = 4$, $x_{26} = 2$, $x_{31} = 1$, $x_{32} = \in$, $x_{33} = 1$, $x_{41} = 3$, $x_{44} = 2$, $x_{45} = 4$ and the optimum (minimum) transportation cost is

$$= \text{Rs. } 9 \times 5 + 3 \times 4 + 5 \times 2 + 6 \times 1 + 5 \times \in + 9 \times 1 + 6 \times 3 + 2 \times 2 + 2 \times 4$$

$$= \text{Rs. } (112 + 5 \in)$$

$$= \text{Rs. } 112 \text{ as } \in \to 0.$$

HW: Solve the Transportation table.

Destination

Source	D1	D2	D3	D4	D5	Supply(S_i)
S1	10	2	3	15	9	35
S2	5	10	15	2	4	40
S3	15	5	14	7	15	20
S4	20	15	13	25	8	30
Demand(D_j):	20	20	40	10	35	

THANK YOU

Unit II

TRANSPORTATION PROBLEM – UNBALANCE & MAXIMIZATION SOLUTION

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PROBLEM 1: Solve the Transportation table.

Destination

		A	B	C	D	Supply
Source	1	11	20	7	8	50
	2	21	16	20	12	40
	3	8	12	18	9	70
Demand :		30	25	35	40	

Solution : Since the total supply ($\sum a_i = 160$) is greater than the total demand ($\sum b_j = 130$), the given problem is an unbalanced transportation problem. To convert this into a balanced one, we introduce a dummy destination E with zero unit transportation costs and having demand equal to 160 – 130 = 30 units.

∴ The given problem becomes

Destination

	A	B	C	D	E	Supply
1	**11**	**20**	**7**	**8**	**0**	**50**
Source 2	**21**	**16**	**20**	**12**	**0**	**40**
3	**8**	**12**	**18**	**9**	**0**	**70**
	30	**25**	**35**	**40**	**30**	**160**

By using VAM the initial solution is as shown in the following table

11	20	7 **35**	8 **15**	0
21	16	20	12 **10**	0 **30**
8 **30**	12 **25**	18	9 **15**	0

∴ The initial transportation cost

= Rs. $7 \times 35 + 8 \times 15 + 12 \times 10 + 0 \times 30 + 8 \times 30 + 12 \times 25 + 9 \times 15$

= Rs. 1160/-

	v1=-1	v2= 3	v3= -1	v4= 0	v5= -12
u1= 8	11 (u+v = 7; d = 4)	20 (u+v = 11; d = 9)	7 **35**	8 **15**	0 (u+v = -4; d = 4)
u2=12	21 (u+v = 11; d = 10)	16 (u+v = 15; d = 1)	20 (u+v = 11; d = 9)	12 **10**	0 **30**
u3= 9	8 **30**	12 **25**	18 (u+v = 8; d = 10)	9 **15**	0 (u+v = -3; d = 3)

Since all $d_{ij} > 0$, the solution under the test is optimal and unique

The optimum (minimum) transportation cost

$$= \text{Rs. } 7 \times 35 + 8 \times 15 + 12 \times 10 + 0 \times 30 + 8 \times 30 + 12 \times 25 + 9 \times 15$$

$$= \text{Rs. } 1160/-$$

PROBLEM 2: Solve the Transportation table.

Destination

		D_1	D_2	D_3	D_4	Supply
	S_1	6	1	9	3	70
Source	S_2	11	5	2	8	55
	S_3	10	12	4	7	70
Demand		85	35	50	45	

Solution : Since the total demand ($\sum b_i = 215$) is greater than the total supply ($\sum a_j = 195$), the given problem is unbalanced transportation problem. To convert this into a balanced one, we introduce a dummy source S_4 with zero unit transportation costs and having supply equal to $215 - 195 = 20$ units. $\therefore$ The given problem becomes

	D_1	D_2	D_3	D_4	Supply
S_1	6	1	9	3	70
S_2	11	5	2	8	55
S_3	10	12	4	7	70
S_4	0	0	0	0	20
	85	35	50	45	215

By VAM method the initial basic feasible solution

6 65	1 5	9	3
11	5 30	2 25	8
10	12	4 25	7 45
0 20	0	0	0

∴ The initial transportation cost

$= \text{Rs. } 6 \times 65 + 1 \times 5 + 5 \times 30 + 2 \times 25 + 4 \times 25 + 7 \times 45 + 0 \times 20$

= Rs. 1010/-

6 **65**	1 **5**	9 −2 [11]	3 1 [2]	$u_1 = 6$
11 10 [1]	5 **30**	2 **25**	8 5 [3]	$u_2 = 10$
10 12 [−2]	12 7 [5]	4 **25**	7 **45**	$u_3 = 12$
0 **20**	0 −5 [5]	0 −8 [8]	0 −5 [5]	$u_4 = 0$
$v_1 = 0$	$v_2 = -5$	$v_3 = -8$	$v_4 = -5$	

6 65 −θ	1 5 +θ	9	3
11	5 30 −θ	2 25 +θ	8
10 +θ	12	4 25 −θ	7 45
0 20	0	0	0

6 (40)	1 (30)	9	3
11	5 (5)	2 (50)	8
10 (25)	12	4	7 (45)
0 (20)	0	0	0

6 40	1 30	9 −2 [11]	3 3 [0]	$u_1 = 6$
11 10 [1]	5 5	2 50	8 7 [1]	$u_2 = 10$
10 25	12 5 [7]	4 2 [2]	7 45	$u_3 = 10$
0 20	0 −5 [5]	0 −8 [8]	0 −3 [3]	$u_4 = 0$
$v_1 = 0$	$v_2 = -5$	$v_3 = -8$	$v_4 = -3$	

Since all $d_{ij} > 0$ with $d_{14} = 0$, the solution under the test is optimal and an alternative optimal solution exists.

The optimum (minimum) transportation cost

= Rs. $6 \times 40 + 1 \times 30 + 5 \times 5 + 2 \times 50 + 10 \times 25 + 7 \times 45 + 0 \times 20$

= Rs. 960/-

PROBLEM 3: Solve the Transportation table.

		Destination			
		D_1	D_2	D_3	Supply (units)
Origin	A	5	6	9	100
	B	3	5	10	75
	C	6	7	6	50
	D	6	4	10	75
Demand (units)		70	80	120	

PROBLEM 4: Solve the Transportation table to Maximization of profit.

Profits (Rs)/Unit

Destination

Source	A	B	C	D	Supply
1	40	25	22	33	100
2	44	35	30	30	30
3	38	38	28	30	70
Demand	40	20	60	30	

Solution: Since the given problem is of maximization type, first convert this into a minimization problem by subtracting the cost elements (entries or c_{ij}) from the highest cost element (c_{ij} = 44) in the given transportation problem. Then the given problem becomes.

Destination

	A	B	C	D	Supply
Source 1	4	19	22	11	100
Source 2	0	9	14	14	30
Source 3	6	6	16	14	70
Demand	40	20	60	30	

This modified minimization problem is unbalanced ($\sum a_i = 200$, $\sum b_j = 150$ and $\sum a_i \neq \sum b_j$). To make it balanced, we introduce a dummy destination E with demand (200 – 150) = 50 units with zero costs c_{ij}. Hence the balanced minimization transportation problem becomes

Destination

	A	B	C	D	E	Supply
Source 1	4	19	22	11	0	100
Source 2	0	9	14	14	0	30
Source 3	6	6	16	14	0	70
Demand	40	20	60	30	50	200

By VAM method the initial basic feasible solution

4 (10)	19	22 (60)	11 (30)	0
0 (30)	9	14	14	0
6	6 (20)	16	14	0 (50)

Since the number of non-negative allocations at independent position is 6, which is less than $(m + n - 1) = (3 + 5 - 1) = 7$, this initial solution is degenerate.

To resolve degeneracy, we allocate a very small quantity $\in$ to the cell (1,5) so that the number of occupied cells becomes $(m + n - 1)$. Hence the initial solution is given by

4 (10)	19	22 (60)	11 (30)	0 (∈)
0 (30)	9	14	14	0
6	6 (20)	16	14	0 (50)

	v1=4	v2=6	v3=22	v4=11	v5=0
u1=0	4 10	19 6 13	22 60	11 30	0 ∈
u2=-4	0 30	9 2 7	14 18 -4	14 7 7	0 -4 4
u3=0	6 4 2	6 20	16 22 * -6	14 11 3	0 50

	v1=4	v2=6	v3=22	v4=11	v5=0
u1=0	4 **10**	19 6 13	22 **60** -	11 **30**	0 **∈** +
u2=-4	0 **30**	9 2 7	14 18 -4	14 7 7	0 -4 4
u3=0	6 4 2	6 **20**	16 22 + * -6	14 11 3	0 **50** -

4 10	19	22 10	11 30	0 50
0 30	9	14	14	0
6	6 20	16 50	14	0

4 **10**	19 12 [7]	22 **10**	11 **30**	0 **50**	$u_1 = 0$
0 **30**	9 8 [1]	14 18 [–4]	14 7 [7]	0 –4 [4]	$u_2 = -4$
6 –2 8	6 **20**	16 **50**	14 5 [9]	0 –6 [6]	$u_3 = -6$
$v_1 = 4$	$v_2 = 12$	$v_3 = 22$	$v_4 = 11$	$v_5 = 0$	

4 20 +θ	19	22 10 −θ	11 30	0 50
0 30 −θ	9	14 +θ	14	0
6	6 20	16 50	14	0

4 20	19	22	11 30	0 50
0 20	9	14 10	14	0
6	6 20	16 50	14	0

4 **20**	19 8 [11]	22 18 [4]	11 **30**	0 **50**	$u_1 = 0$
0 **20**	9 4 [5]	14 **10**	14 7 [7]	0 –4 [4]	$u_2 = -4$
6 2 [4]	6 **20**	16 **50**	14 9 [5]	0 –2 [2]	$u_3 = -2$
$v_1 = 4$	$v_2 = 8$	$v_3 = 18$	$v_4 = 11$	$v_5 = 0$	

Since all $d_{ij} > 0$, the current solution is optimal and unique.

The optimum profit

= Rs. $40 \times 20 + 33 \times 30 + 0 \times 50 + 44 \times 20 + 30 \times 10 + 38 \times 20 + 28 \times 50$

= Rs. 5130/-.

PROBLEM 5: Solve the Transportation table to Maximization of profit.

Destination

	A	B	C	D	Supply
Source 1	15	51	42	33	23
Source 2	80	42	26	81	44
Source 3	90	40	66	60	33
Demand	23	31	16	30	100

THANK YOU

Unit II

TRANSPORTATION ALGORITHM (OR) MODI METHOD - TEST FOR OPTIMAL SOLUTION

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MODI Method - MOdified DIstribution Method

Step 1: Find the initial basic feasible solution of the given problem by Northwest Corner Rule (or) Least Cost Method (or) VAM.

Step 2: Check the number of occupied cells. If there are less than $m + n - 1$, there exists degeneracy and we introduce a very small positive assignment of in suitable independent positions, so that the number of occupied cells is exactly equal to $m + n - 1$.

Step 3: Find the set values u_i, v_j ($i = 1, 2, \ldots m$; $j = 1, 2, \ldots n$) from the relation $c_{ij} = u_i + v_j$ for each occupied cell (i, j), by starting initially with $u_i = 0$ or $v_j = 0$ preferably for which the corresponding row or column has maximum number of individual allocations.

Step 4: Find $u_i + v_j$ for each unoccupied cell (i, j) and enter at the upper right corner of the corresponding cell (i, j).

Step 5: Find the cell evaluations $d_{ij} = c_{ij} – (u_i + v_j)$ for each unoccupied cell (i, j) and enter at the upper left corner of the corresponding cell (i, j).

Step 6: Examine the cell evaluations d_{ij} for all unoccupied cells (i, j) and conclude that,

•If all $d_{ij} > 0$, then the solution under the test is optimal and unique.

•If all $d_{ij} > 0$, with at least one $d_{ij} = 0$, then the solution under the test is optimal and an alternative optimal solution exists.

•If at least one $d_{ij} < 0$, then the solution is not optimal. Go to the next step.

Step 7: Form a new BFS by giving maximum allocation to the cell for which d_{ij} is most negative by making an occupied cell empty. For that draw a closed path consisting of horizontal and vertical lines beginning and ending at the cell for which d_{ij} is most negative and having its other corners at some allocated cells. Along this closed loop indicate +θ and –θ alternatively at the corners. Choose minimum of the allocations from the cells having –θ. Add this minimum allocation to the cells with +θ and subtract this minimum allocation from the allocation to the cells with –θ.

Step 8: Repeat steps (2) to (6) to test the optimality of this new basic feasible solution.

Step 9: Continue the above procedure till an optimum solution is attained.

PROBLEM 1: Solve the Transportation table.

		Destination				
		D1	D2	D3	D4	Supply(S_i)
Source	O1	3	1	7	4	250
	O2	2	6	5	9	350
	O3	8	3	3	2	400
Demand(D_j):		200	300	350	150	

Solution:

Step 1: Check whether the problem is balanced or not.

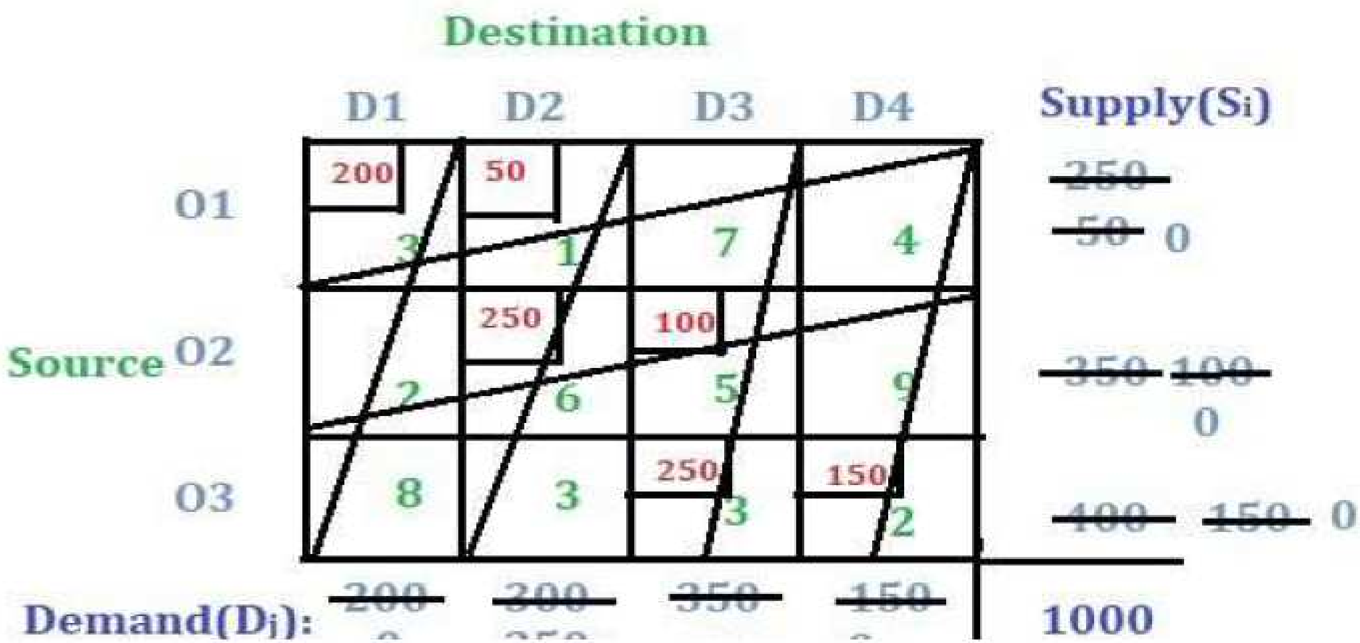
If the total sum of all the supply from sources **O1**, **O2**, and **O3** is equal to the total sum of all the demands for destinations **D1**, **D2**, **D3** and **D4** then the transportation problem is a balanced transportation problem.

Destination

Source		D1	D2	D3	D4	Supply(S_i)
	O1	3	1	7	4	250
Source	O2	2	6	5	9	350
	O3	8	3	3	2	400
Demand(D_j):		200	300	350	150	1000

Step 2: Finding the initial basic feasible solution.

Any of the three aforementioned methods can be used to find the initial basic feasible solution. Here, NorthWest Corner Method will be used. And according to the NorthWest Corner Method this is the final initial basic feasible solution:



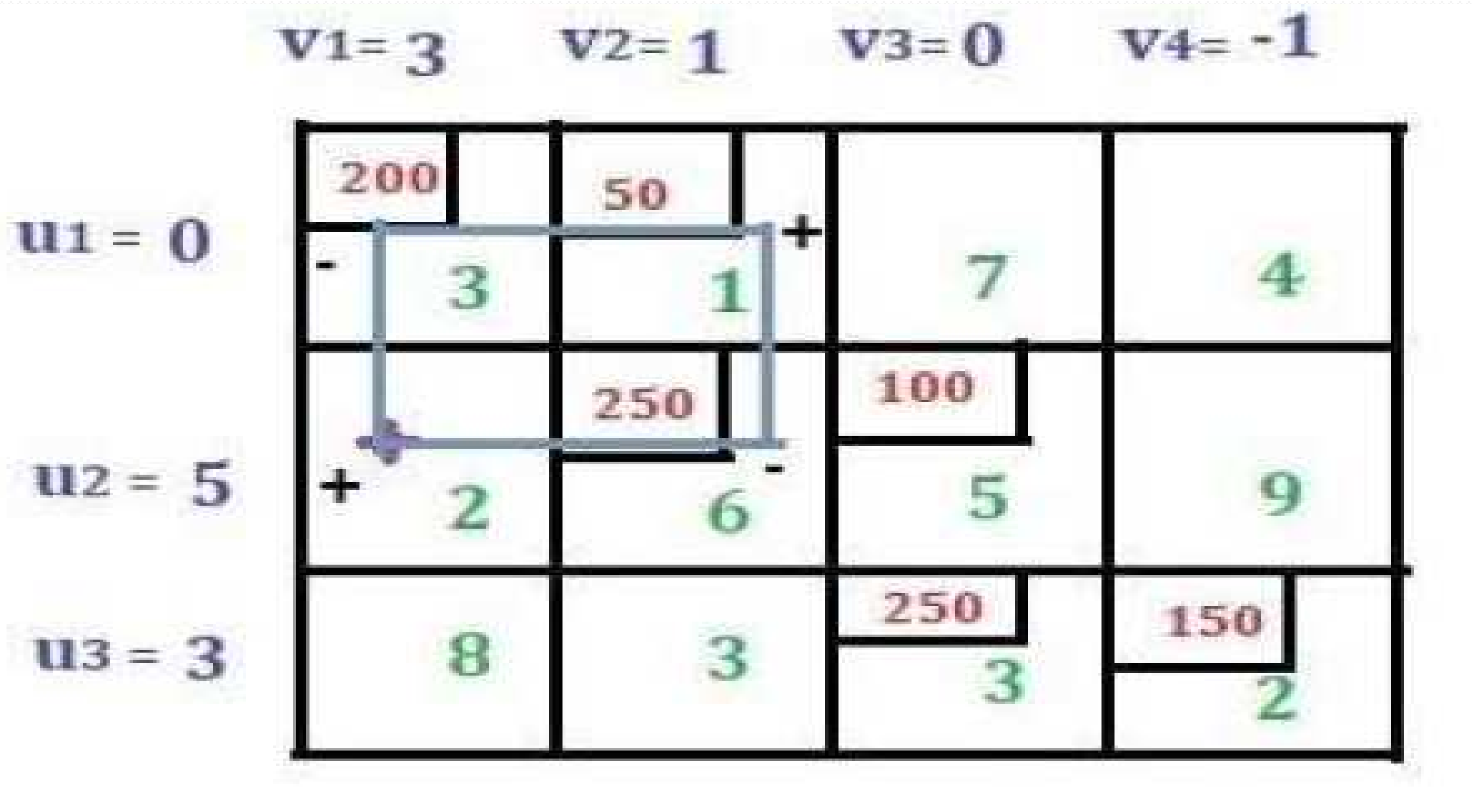
Now, the total cost of transportation will be **(200 * 3) + (50 * 1) + (250 * 6) + (100 * 5) + (250 * 3) + (150 * 2) = 3700**.

Find the optimal Solution :

- Find set values u_i, v_j (i = 1, 2, …m ; j = 1, 2, …n) from the relation $c_{ij} = u_i + v_j$ for each occupied cell (i, j),
- Find $u_i + v_j$ for each unoccupied cell (i, j) and enter at the upper right corner
- Find the cell evaluations $d_{ij} = c_{ij} – (u_i + v_j)$ for each unoccupied cell (i, j) and enter at the lower left corner of the corresponding cell (i, j).
- If all $d_{ij} > 0$, then the solution under the test is optimal and unique.
- If all $d_{ij} > 0$, with at least one $d_{ij} = 0$, then the solution under the test is optimal and an alternative optimal solution exists.
- If at least one $d_{ij} < 0$, then the solution is not optimal.

	V1= 3	V2= 1	V3= 0	V4= -1
u1 = 0	200 / 3	50 / 1	0 / 7 / 7	-1 / 5 / 4
u2 = 5	8 / + / -6 / 2	250 / 6	100 / 5	4 / 5 / 9
u3 = 3	6 / 8 / 2	4 / 3 / -1	250 / 3	150 / 2

Form a new BFS by giving maximum allocation to the cell for which d_{ij} is most negative by making an occupied cell empty. For that draw a closed path consisting of horizontal and vertical lines beginning and ending at the cell for which d_{ij} is most negative and having its other corners at some allocated cells. Along this closed loop indicate $+\theta$ and $-\theta$ alternatively at the corners. Choose minimum of the allocations from the cells having $-\theta$.



Add this minimum allocation to the cells with +θ and subtract this minimum allocation from the allocation to the cells with –θ.

3	250 1	7	4
200 2	50 6	100 5	9
8	3	250 3	150 2

	V1= 2	V2= 6	V3= 5	V4= 4
u1=-5	-3 6 3	250 1	0 7 7	-1 5 4
u2= 0	200 2	50 6	100 5	4 5 9
u3=-2	0 8 8	4 3 -1	250 3	150 2

3	(250) 1	7	4
(200) 2	(50) - 6	(100) +5	9
8	+ 3	(250) - 3	(150) 2

3	(250) 1	7	4
(200) 2	6	(150) 5	9
8	(50) 3	(200) 3	(150) 2

	V1= 0	V2=3	V3=3	V4= 2
u1=-2	-2 5 3	250 1	1 6 7	0 4 4
u2=2	200 2	5 1 6	150 5	4 5 9
u3=0	0 8 8	50 3	200 3	150 2

Here all $d_{ij} > 0$, then the solution under the test is optimal and unique.

the total cost i.e. $(250 * 1) + (200 * 2) + (150 * 5) + (50 * 3) + (200 * 3) + (150 * 2) = 2450$

THANK YOU

Unit II

ASSIGNMENT PROBLEM

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- An assignment problem is a particular case of transportation problem where the objective is to assign a number of resources to an equal number of activities so as to minimize total cost or maximize total profit of allocation.

- The problem of assignment arises because available resources such as men, machines etc. have varying degrees of efficiency for performing different activities, therefore, cost, profit or loss of performing the different activities is different.

- Suppose that we have '*n*' jobs to be performed on '*m*' machines (one job to one machine) and our objective is to assign the jobs to the machines at the minimum cost (or maximum profit) under the assumption that each machine can perform each job but with varying degree of efficiencies.

- The assignment problem can be stated that in the form of m x n matrix (c_{ij}) called a ***Cost Matrix*** (or) ***Effectiveness Matrix*** where c_{ij} is the cost of assigning i^{th} machine to j^{th} job.

Jobs

		1	2	3	…	…	n
Machines	1	c_{11}	c_{12}	c_{13}	⋯	⋯	c_{1n}
	2	c_{21}	c_{22}	c_{23}	⋯	⋯	c_{2n}
	3	c_{31}	c_{32}	c_{33}	⋯	⋯	c_{3n}
	⋮	⋮	⋮	⋮	…	…	⋮
	⋮	⋮	⋮	⋮	…	…	⋮
	m	c_{m1}	c_{m2}	c_{m3}	⋯	⋯	c_{mn}

MATHEMATICAL FORMULATION OF AN ASSIGNMENT PROBLEM

Consider an assignment problem of assigning n jobs to n machines (one job to one machine). Let c_{ij} be the unit cost of assigning i^{th} machine to the j^{th} job and,

$$\text{Let } x_{ij} = \begin{cases} 1, & \text{if } j^{th} \text{ job is assigned to } i^{th} \text{ machine} \\ 0, & \text{if } j^{th} \text{ job is not assigned to } i^{th} \text{ machine} \end{cases}$$

The assignment model is then given by

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \text{ subject to the constraints,}$$

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1,2, \cdots n$$

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2, \cdots n$$

$$x_{ij} = 0 \ (or) \ 1.$$

DIFFERENCE BETWEEN TRANSPORTATION PROBLEM AND ASSIGNMENT PROBLEM

Transportation Problem	Assignment Problem
a. Supply at any source may be any positive quantity a_i.	Supply at any source (machine) will be 1. i.e., $a_i = 1$.
b. Demand at any destination may be any positive quantity b_j.	Demand at any destination (job) will be 1. i.e., $b_j = 1$.
c. One or more source to any number of destinations.	One source (machine) to only one destination (job).

ASSIGNMENT ALGORITHM (OR) HUNGARIAN METHOD

First check whether the number of rows is equal to number of columns, if it is so, the assignment problem is said to be balanced. Then proceed to step 1. If it is not balanced, then it should be balanced before applying the algorithm.

Step 1: Subtract the smallest cost element of each row from all the elements in the row of the given cost matrix. See that each row contains at least one zero.

Step 2: Subtract the smallest cost element of each column from all the elements in the column of the resulting cost matrix obtained by step 1 and make sure each column contains at least one zero.

Step 3: (Assigning the zeros)

(a) Examine the rows successively until a row with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it. Cross all other zeros in the column of this encircled zero, as these will not be considered for any future assignment. Continue in this way until all the rows have been examined.

(b) Examine the columns successively until a column with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it and cross any other zero in its row. Continue until all the columns have been examined.

Step 4: (Apply Optimal Test)

(a) If each row and each column contain exactly one encircled zero, then the current assignment is optimal.

(b) If at least one row / column is without an assignment (i.e., if there is at least one row / column is without one encircled zero), then the current assignment is not optimal. Go to step 5.

Step 5: Cover all the zeros by drawing a minimum number of straight lines as follows:

(a) Mark the rows that do not have assignment.

(b) Mark the columns (not already marked) that have zeros in marked rows.

(c) Mark the rows (not already marked) that have assignments in marked columns.

(d) Repeat (*b*) and (*c*) until no more marking is required.

(e) Draw lines through all unmarked rows and marked columns. If the number of these lines is equal to the order of the matrix then it is an optimum solution otherwise not.

Step 6: Determine the smallest cost element not covered by the straight lines. Subtract this smallest cost element from all the uncovered elements and add this to all those elements which are lying in the intersection of these straight lines and do not change the remaining elements which lie on the straight lines.

Step 7: Repeat steps (1) to (6), until an optimum assignment is obtained.

ASSIGNMENT PROBLEM HUNGARIAN METHOD - EXAMPLE

PROBLEM 1: Solve the following assignment problem shown in Table using Hungarian method.

The matrix entries are processing time of each man in hours.

		Men				
		1	**2**	**3**	**4**	**5**
	I	20	15	18	20	25
	II	18	20	12	14	15
Job	**III**	21	23	25	27	25
	IV	17	18	21	23	20
	V	18	18	16	19	20

Solution: The given problem is balanced with 5 job and 5 men

$$\text{Let} \quad A = \begin{pmatrix} 20 & 15 & 18 & 20 & 25 \\ 18 & 20 & 12 & 14 & 15 \\ 21 & 23 & 25 & 27 & 25 \\ 17 & 18 & 21 & 23 & 20 \\ 18 & 18 & 16 & 19 & 20 \end{pmatrix}$$

$$\simeq \begin{pmatrix} 5 & 0 & 3 & 5 & 10 \\ 6 & 8 & 0 & 2 & 3 \\ 0 & 2 & 4 & 6 & 4 \\ 0 & 1 & 4 & 6 & 3 \\ 2 & 2 & 0 & 3 & 4 \end{pmatrix}$$

$$\simeq \begin{pmatrix} 5 & \boxed{0} & 3 & 3 & 7 \\ 6 & 8 & \cancel{0} & \boxed{0} & \cancel{0} \\ \boxed{0} & 2 & 4 & 4 & 1 \\ \cancel{0} & 1 & 4 & 4 & \boxed{0} \\ 2 & 2 & \boxed{0} & 1 & 1 \end{pmatrix}$$

Therefore, the optimal solution is:

Job	Men	Time
I	2	15
II	4	14
III	1	21
IV	5	20
V	3	16
	Total time =	86 hours

PROBLEM: 2 The processing time in hours for the jobs when allocated to the different machines are indicated below. Assign the machines for the jobs so that the total processing time is minimum

		Machines				
		M_1	M_2	M_3	M_4	M_5
	J_1	9	22	58	11	19
	J_2	43	78	72	50	63
Jobs	J_3	41	28	91	37	45
	J_4	74	42	27	49	39
	J_5	36	11	57	22	25

Solution: The given problem is balanced with 5 job and 5 machines

$$\text{Let } A = \begin{pmatrix} 9 & 22 & 58 & 11 & 19 \\ 43 & 78 & 72 & 50 & 63 \\ 41 & 28 & 91 & 37 & 45 \\ 74 & 42 & 27 & 49 & 39 \\ 36 & 11 & 57 & 22 & 25 \end{pmatrix}$$

$$A \simeq \begin{pmatrix} 0 & 13 & 49 & 2 & 10 \\ 0 & 35 & 29 & 7 & 20 \\ 13 & 0 & 63 & 9 & 17 \\ 47 & 15 & 0 & 22 & 12 \\ 25 & 0 & 46 & 11 & 14 \end{pmatrix} \simeq \begin{pmatrix} 0 & 13 & 49 & 0 & 0 \\ 0 & 35 & 29 & 5 & 10 \\ 13 & 0 & 63 & 7 & 7 \\ 47 & 15 & 0 & 20 & 2 \\ 25 & 0 & 46 & 9 & 4 \end{pmatrix}$$

$$\simeq \begin{pmatrix} \cancel{0} & 13 & 49 & \boxed{0} & \cancel{0} \\ \boxed{0} & 35 & 29 & 5 & 10 \\ 13 & \boxed{0} & 63 & 7 & 7 \\ 47 & 15 & \boxed{0} & 20 & 2 \\ 25 & \cancel{0} & 46 & 9 & 4 \end{pmatrix} \begin{matrix} \\ \\ \checkmark \\ \\ \checkmark \end{matrix} \qquad \simeq \begin{pmatrix} \cancel{0} & 13 & 49 & \boxed{0} & \cancel{0} \\ \boxed{0} & 35 & 29 & 5 & 10 \\ 13 & \boxed{0} & 63 & 7 & 7 \\ 47 & 15 & \boxed{0} & 20 & 2 \\ 25 & \cancel{0} & 46 & 9 & 4 \end{pmatrix} \begin{matrix} \\ \\ \checkmark \\ \\ \checkmark \end{matrix}$$

$$\begin{matrix} & \checkmark & & & \end{matrix} \qquad\qquad \begin{matrix} & \checkmark & & & \end{matrix}$$

$$\simeq \begin{pmatrix} 0 & 17 & 49 & 0 & 0 \\ 0 & 39 & 29 & 5 & 10 \\ 9 & 0 & 59 & 3 & 3 \\ 47 & 19 & 0 & 20 & 2 \\ 21 & 0 & 42 & 5 & 0 \end{pmatrix}$$

$$\simeq \begin{pmatrix} \cancel{0} & 17 & 49 & \boxed{0} & \cancel{0} \\ \boxed{0} & 39 & 29 & 5 & 10 \\ 9 & \boxed{0} & 59 & 3 & 3 \\ 47 & 19 & \boxed{0} & 20 & 2 \\ 21 & \cancel{0} & 42 & 5 & \boxed{0} \end{pmatrix}$$

$\therefore$ The optimum assignment schedule is $J_1 \rightarrow M_4$, $J_2 \rightarrow M_1$, $J_3 \rightarrow M_2$, $J_4 \rightarrow M_3$, $J_5 \rightarrow M_5$ and the optimum (minimum) processing time

$= 11 + 43 + 28 + 27 + 25$ hours

$= 134$ hours.

PROBLEM 3: At the head office of a company there are five registration counters. Five persons are available for service. How should the counters be assigned to persons so as to **maximize the profit**?

	Person				
Counter	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

Solution: The given problem is balanced with 5 counter and 5 person

$$A = \begin{pmatrix} 30 & 37 & 40 & 28 & 40 \\ 40 & 24 & 27 & 21 & 36 \\ 40 & 32 & 33 & 30 & 35 \\ 25 & 38 & 40 & 36 & 36 \\ 29 & 62 & 41 & 34 & 39 \end{pmatrix}$$

To convert the problem as minimization we reduce the matrix by subtracting all entry by the largest value , that is 62

$$A = \begin{pmatrix} 32 & 25 & 22 & 34 & 22 \\ 22 & 38 & 35 & 41 & 26 \\ 22 & 30 & 29 & 30 & 27 \\ 37 & 24 & 22 & 26 & 26 \\ 33 & 0 & 21 & 28 & 23 \end{pmatrix}$$

A =

10	3	0	8	~~0~~
0	16	13	15	4
~~0~~	8	7	6	5
15	2	~~0~~	0	4
33	0	21	24	23

A =

10	3	0	8	~~0~~
0	16	13	15	4
~~0~~	8	7	6	5
15	2	~~0~~	0	4
33	0	21	24	23

A =

14	3	0	8	0
0	12	9	11	0
0	4	3	2	1
19	2	0	0	4
37	0	21	24	23

Optimal Assignment Root= 1-C , 2-E, 3A + 4D + 5B

Substituting values from original table: 40 + 36 + 40 + 36 + 62 = 214.

PROBLEM: 4 **A company has four machines to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table.**

		Machines			
		1	2	3	4
	A	18	24	28	32
Jobs	B	8	13	17	19
	C	10	15	19	22

What are job assignments which will minimize the cost?

Solution:

Since the number of rows is less than the number of columns in the cost matrix, the given assignment problem is unbalanced.

To make it a balanced one, add a dummy job D (row) with zero cost elements. The balanced cost matrix is given by

Let $A = \begin{pmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$$\simeq \begin{pmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\simeq \begin{pmatrix} \boxed{0} & 6 & 10 & 14 \\ \cancel{0} & 5 & 9 & 11 \\ \cancel{0} & 5 & 9 & 12 \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{pmatrix}$$

$$\simeq \begin{pmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\simeq \begin{pmatrix} 0 & 1 & 5 & 9 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 4 & 7 \\ 5 & 0 & 0 & 0 \end{pmatrix}$$

$$\simeq \begin{pmatrix} 0 & 1 & 5 & 9 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 4 & 7 \\ 5 & 0 & 0 & 0 \end{pmatrix}$$

$$\cong \begin{pmatrix} 0 & 1 & 5 & 9 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 4 & 7 \\ 5 & 0 & 0 & 0 \end{pmatrix}$$

$$\cong \begin{pmatrix} 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 9 & 4 & 0 & 0 \end{pmatrix}$$

$$\cong \begin{pmatrix} 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 9 & 4 & 0 & 0 \end{pmatrix}$$

$\therefore$ The optimum assignment schedule is given by $A \to 1$, $B \to 2$, $C \to 3$, $D \to 4$ and the optimum (minimum) assignment cost $= (18 + 13 + 19 + 0)$ cost units $=$ 50/- units of cost

Travelling Salesman Problem

A travelling salesman plans to visit n cities. He wishes to visit each city only once, and again arriving back to his home city from where he started. So that the total travelling distance is minimum.

If there are n cities, then there are (n - 1)! possible ways for his tour. For example, if the number of cities to be visited is 4, then there are 3! = 6 different combination is possible. Such type of problems can be solved by Hungarian method, branch and bound method, penalty method, nearest neighbor method.

Example 1:

A travelling salesman has to visit four cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling time (in hours) for each city from a particular city is given below:

$$\text{From}\quad \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\!\!\begin{array}{c} \text{To} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} - & 46 & 16 & 40 \\ 41 & - & 50 & 40 \\ 82 & 32 & - & 60 \\ 40 & 40 & 36 & - \end{pmatrix} \end{array}$$

Solution:

The Cost matrix of the given travelling salesman problem is

STEP1:

$$\begin{pmatrix} \infty & 46 & 16 & 40 \\ 41 & \infty & 50 & 40 \\ 82 & 32 & \infty & 60 \\ 40 & 40 & 36 & \infty \end{pmatrix}$$

Now we solve this as a routine assignment problem

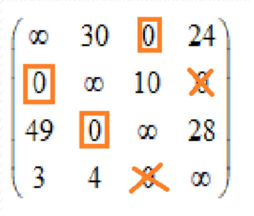
STEP 2:

$$\begin{pmatrix} \infty & 30 & 0 & 24 \\ 1 & \infty & 10 & 0 \\ 50 & 0 & \infty & 28 \\ 4 & 4 & 0 & \infty \end{pmatrix}$$

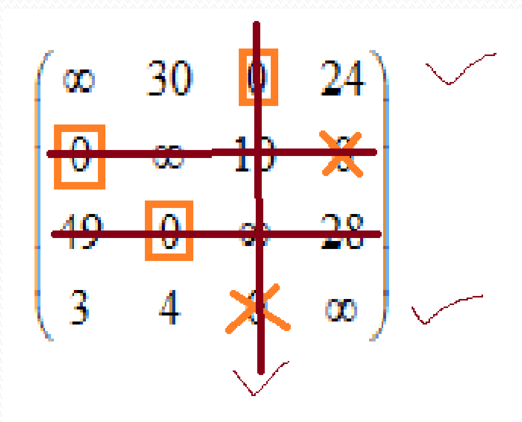
STEP 3:

$$\begin{pmatrix} \infty & 30 & 0 & 24 \\ 0 & \infty & 10 & 0 \\ 49 & 0 & \infty & 28 \\ 3 & 4 & 0 & \infty \end{pmatrix}$$

STEP 4:



STEP 5:



STEP 6:

$$\begin{pmatrix} \infty & 27 & 0 & 21 \\ 0 & \infty & 13 & 0 \\ 49 & 0 & \infty & 28 \\ 0 & 1 & 0 & \infty \end{pmatrix}$$

STEP 8:

$$\begin{pmatrix} \infty & 27 & \boxed{0} & 21 \\ \cancel{0} & \infty & 13 & \boxed{0} \\ 49 & \boxed{0} & \infty & 28 \\ \boxed{0} & 1 & \cancel{0} & \infty \end{pmatrix}$$

Since each row and each column contains exactly one encircled zero, the current assignment is optimal for the assignment problem.

Therefore the optimum assignment schedule is given by

$$A \to C,\ \ B \to D,\ C \to B,\ D \to A$$

(i.e) $$A \to C,\ \ C \to B,\ B \to D,\ D \to A$$

(i.e) $$A \to C \to B \to D \to A$$

Therefore the required minimum costs = 16+32+40+40

= 128/- units of cost.

THANK YOU